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Exam 1

You are allowed a non-graphing calculator without CAS (computer algebra system). You are also allowed both sides of a standard sized printer paper (like all the worksheets) as an equation sheet. This sheet must be handwritten (with pen or pencil). It cannot be typeset or have printed equations. Your equation sheet will be turned in with the exam, so it must have your name in the top left corner.

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dot = 0

1. Let $v = \langle 0, 1, 2 \rangle$. -2

(a) What are two vectors orthogonal to v?

(b) What is a vector parallel to v which has magnitude 3?

scalar

a) ~~$\vec{w} = \langle 0,0,0 \rangle$, $\vec{a} = \langle 10,0,0 \rangle$ $\vec{w} = \langle 0,2,4 \rangle$~~

~~$\vec{v} = \langle 0,-1,-2 \rangle$~~

b) mag of 3 → $9 = x^2+y^2+z^2$ or $\sqrt{81}$, or $9 = x^2+y^2$, parallel

$\vec{b} \neq \langle 0, 3 \rangle$ $3 = \sqrt{x^2+y^2}$ ~~$9 = x^2$~~ ~~$9 - y^2 = x^2$~~

scalar

dot = |a||b|

$\sqrt[3]{81}$

2 3, ?

$x^2 + y^2 = 9$

$x + y = 3$

~~$9 = 9 - y^2 + y^2$~~

a) $\vec{w} = \langle 0, 2, -1 \rangle$

$\boxed{\vec{w} = \langle 0, 2, -1 \rangle \quad \vec{v} = \langle 0, -1, 1/2 \rangle}$

$\boxed{\text{a) } \vec{b} = \langle 0, -1, 1/2 \rangle}$

$\boxed{\vec{w} = \langle 0, 4, -2 \rangle}$ //

$\boxed{\text{b) } \vec{r} = \langle 0, 1.5, 1.5 \rangle}$

parallel?

2 4 4 16

4 8

8 16

16 32

-Um

2. Let $P_1 = (1,1,1)$, $P_2 = (2,2,2)$, and $P_3 = (1,0,0)$.

(a) What is the area of the resulting triangle?

(b) What is the equation of the plane containing these points?

a) obtain 2 vect: use area of parallelogram

$\vec{v} = \langle 1,1,1 \rangle$ → $\frac{1}{2}|a \times b|$

$\vec{w} = \langle \cancel{3}-1,-2,-2 \rangle$ $\sqrt{}$

$\vec{v} \times \vec{w} \to \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix} \to \cancel{2+1} = \cancel{4}6$

$\cancel{x}$ $2+2+2 \to \cancel{8}6$

$\vec{v} \times \vec{w} = |\langle 2,2,2 \rangle| = \sqrt{4+4+4} = \boxed{\frac{\sqrt{12}}{2}}$

b) $\boxed{2(x-1)+2(y-1)+2(z-1)=0}$

3. Let $\vec{v} = \langle 3, 1 \rangle$.

(a) Create an orthonormal coordinate system with one vector parallel to $\vec{v}$.

(b) Write $\hat{\imath}$ as a linear combination of your coordinate system vectors.

ONCS:

- $norm = |v| = \sqrt{9+1} = \sqrt{10}$

$\Rightarrow \vec{n} = \langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \rangle$

- some perpendic vect: $\vec{p} = \langle 1, -3 \rangle$ $|\vec{p}| = \sqrt{9+1}$

→ normalize → $\langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \rangle$

$$\rightarrow \boxed{\begin{array}{l} \langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \rangle \\ \langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \rangle \end{array}}$$

b) $\hat{\imath} = c_1 \begin{bmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix} + c_2 \begin{bmatrix} \frac{1}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} \end{bmatrix}$

$c_1 \, 3/\sqrt{10} + c_2 \frac{1}{\sqrt{10}}$

$$\rightarrow \begin{pmatrix} \vec{n} \, 3/\sqrt{10} + \vec{p} \, 1/\sqrt{10} \\ \vec{n} \, 1/\sqrt{10} + \vec{p} \, -3/\sqrt{10} \end{pmatrix}$$

4. Consider the curve $r = \sin(\theta)$.

(a) Following polar to cartesian conversions, what is $X(\theta)$?

(b) Following polar to cartesian conversions, what is $Y(\theta)$?

(c) What is the slope as a function of θ, $\frac{dx}{dy}(\theta)$?

(d) What (if any) are the singular points of this parametric curve?

polar → cart

a) $X(\theta) = 0$, since the curve in polar is defined as such.

b) $Y(\theta) = 1$

c) slope = $\frac{?}{h}$ →

d) Take $r'(t) = 0$, solve for t's

?,?

5. Let $P = (0,0)$ and $f(x,y) = \dfrac{x^2y^2}{x^4+y^4}$.

(a) What are two distinct parametric lines passing through P: r_1 and r_2?

(b) What is the limit of f along r_1 at P?

a) $\vec{r}_1 = L(t) = t\langle 1,1\rangle$, given $t_0 = (0,0)$

$\vec{r}_2 = L(t) = t\langle \sin t, \sin t\rangle$, given $t_0 = (0,0)$

b) $r_1 = P = t_0 = 0 \to 1$ for $\langle 0 \rangle$

$\to \lim_{x\to 1}\left(\frac{t^4}{t^8} = \frac{t}{t^2}\right) \to \boxed{\frac{1}{2}}$

need find bounds?

6. Let $r(t) = \langle 3\sin(t), 3\cos(t), 4t\rangle$.

(a) Set up the integral for arclength between $t = 0$ and $t = 1$.

Formula: $\int_a^b |r'(t)|\,dt$,

~~1) Find $\int$ bounds; ? of r = pt, then~~

$\int_0^1$

Let $r'(t) = \langle 3\cos t, -3\sin t, 4\rangle$, $|r'(t)| = (9\cos^2 t + 9\sin^2 t + 16)^{1/2}$

so $\boxed{\int_0^1 |r'(t)|\,dt}$ where $|r'(t)| =$

Come Back

7. Let $P = (3, -4)$ and $f(x, y) = x^2 + y + 1$.

(a) Create a parametric curve which passes through P and is <u>not</u> a line.

(b) Find the derivative of f along r at P.

a) Let $\vec{v} = \begin{cases} x = 3 + \cos t \\ y = -4 + \cos t \end{cases}$ or $\boxed{\langle \cos t, \cos t \rangle + (3, -4)}$

~~$\langle \sin t, \sin t \rangle + (3, 4)$~~

b) Formula:

$\frac{d}{dt}\left(f(r(t))\right]_{t_0}$ or $\frac{1}{f'(t_0)}$

-plug P into r(t) for each x and y of t_0

~~1) $f(r(t)) = 9 - 4 + 1$~~

assume $\vec{r} = \vec{v}$ found in @

Then, ~~$f(x, y) = (3 + \cos t)^2 + (\cos t)$~~

$f(x, y) = (3 + \cos t)^2 - 4 + \cos t + 1$

~~@ t_0, $t = 0 \rightarrow 16 - 2 = 14$~~

$t_0 \rightarrow (3 + \cos(3))^2 - 4\cos(3) + 1$

$f'(t) = 2(3 + \cos t)(-\sin t) - \sin t$, @ t_0 ?

$t_0 \rightarrow (3 + \cos(-4))^2 - 4\cos(-4) + 1$

~~$2(3 + 1)$~~

$\rightarrow f'(t_0) = \boxed{\begin{matrix} t = 3 \\ t = -4 \end{matrix}}$